CONTRIBUTION OF VIABILITY THEORY AND CONSTRAINED OPTIMAL CONTROL TO FERTIGATION

Mahugnon G. Dadjo[1,3], Alain Rapaport[1] , Denis Efimov[2] ,

Jérôme Harmand[3] and Rosane Ushirobira[2]

[1] *MISTEA, Univ. Montpellier, Institut Agro, Montpellier, France*

[2] *Inria, Univ. Lille, CNRS, 59000 Lille, France*

[3] *LBE-INRAE, Univ. Montpellier, Narbonne, France*

ABSTRACT

In the prevailing context of climate change, water resources are subjected to mounting pressures, not only in southern nations but also across Europe. The adaptation of societies to these global transformations necessitates a comprehensive overhaul of resource management strategies, coupled with the implementation of soberness measures. Agriculture, as the most water-intensive human activity, accounts for nearly 40% of water withdrawals in Europe. Consequently, the optimization of irrigation emerges as a paramount objective for the future of agriculture.

Over the past several decades, agronomists have developed models to enhance our understanding of crop growth dynamics, its prediction and to optimize agricultural production. Crops need nutrients that can be present in soil or brought as fertilizers. The question of interest here is the fertigation for which nutrients are brought with the irrigated water. Initially concentrated on water supply, these models have progressively integrated nitrogen dynamics [4]. The application of such models to optimize the water and nutrient supply to crops is essential for curtailing the use of synthetic inputs and minimizing water consumption in agriculture ensuring a good crop production, particularly within the framework of reuse water, where the goal is to diminish or eliminate the reliance on chemical fertilizers.

This paper presents a simplified crop model that encapsulates both water and nitrogen stress, offering a valuable tool for irrigation decision-making.

Model description : Crop model describes the interaction between soil moisture $S(t)$, total soil nitrogen content $N(t)$ and biomass production $B(t)$. The dynamics are as follows :

. Soil water balance : The relative soil humidity in the root zone (dimensionless between 0 and 1) is modeled as a balance between gains from irrigation flow rate u and losses mainly due to soil evaporation $(1-\varphi(t))K_R(S)$, crop transpiration $\varphi(t)K_S(S)$ as given in equation (1)

$$\frac{dS}{dt}=k_1\left(-\varphi(t)K_S(S)-(1-\varphi(t))K_R(S)+k_2u(t)\right) \quad (1)$$

. Soil nitrogen balance : The variation of the nitrogen is the compensation of the plant uptake of nitrogen by the nitrogen of irrigation water which as indicated in equation (2)

$$\frac{dN}{dt}=-k_3\varphi(t)K_S(S)f\left(\frac{N}{S}\right)+k_4C_N^{in}u(t) \quad (2)$$

. Crop biomass : The model assumes that biomass production is proportional to nitrogen uptake, and it dynamic is described as $\frac{dB}{dt}=\varphi(t)K_S(S)f\left(\frac{N}{S}\right)$ (3)

where the function $\varphi(t)$ represents the vegetal cover at time t (related to the LAI, Leaf Area Indice) the function K_S (see Fig.a) is used to capture the plant stomatal response to soil moisture condition, the function K_R (see Fig.b) reduce evaporation according to soil humidity and the function f (see Fig.c) limits the nitrogen uptake above a certain critical concentration η_c and are given by

$$K_S(S)=\begin{cases} 0, \text{ if } S\leq S_w \\ \frac{S-S_w}{S^*-S_W}, \text{ if } S_w<S\leq S^* \\ 1, \text{ if } S>S^* \end{cases} \; ; \; K_R(S)=\begin{cases} 0, \text{ if } S\leq S_h \\ \frac{S-S_h}{1-S_h}, \text{ if } S>S_h \end{cases} \text{ and } f\left(\frac{N}{S}\right)=\begin{cases} \frac{N}{\eta_c S}, \text{ if } 0\leq\frac{N}{S}\leq\eta_c \\ 1, \text{ if } \frac{N}{S}\geq\eta_c \end{cases}$$

where S_w is the wilting point, S^* is the point of incipient stomatal closure and S_h the hygroscopic point. In the model, the parameter k_1 is associated with the porosity of the soil, indicating how porous the soil is and thereby influencing the movement and retention of substances within it. Parameters k_2, k_3, k_4 are conversion coefficients, which likely adjust the model to account for various transformations or interactions within the system. C_N^{in} represents the concentration of nitrogen present in the (treated) irrigated wastewater. Moreover, we say that crops suffer water stress when the function K_S is not maximum i.e $S < S^*$ and they suffer nitrogen stress when the function f is not maximum i.e $\frac{N}{S}<\eta_c$.

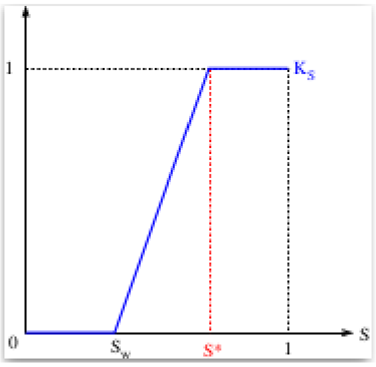

Fig.a

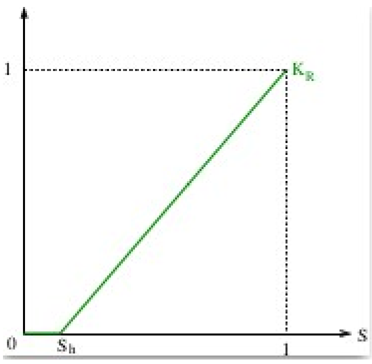

Fig.b

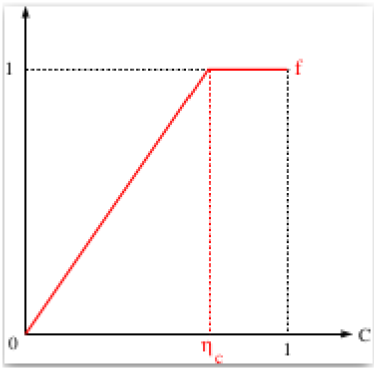

Fig.c

We assume that other essential resources for crop growth, such as phosphorus and minerals, are not limiting factors. The distinguishing feature of this model, in contrast to those found in existing literature, lies in its explicit consideration of nitrogen supply through irrigation, a process known as "fertigation". Depending on the nitrogen concentration in the irrigation water, a dilution effect may occur, reducing the available nitrogen for the crop, thereby exacerbating nitrogen stress while simultaneously lowering costs, even as water stress is avoided. The objective of this study is to leverage the model to identify the optimal balance between water and nitrogen inputs necessary for achieving robust growth performance. Fertigation is generally employed when soil nitrogen content is inadequate to ensure maximum growth; this deficiency can be offset by the nitrogen present in the irrigation water. Both the initial nitrogen concentration in the soil and the nitrogen concentration in the irrigation water play pivotal roles. To determine the possibilities of ensuring a given biomass production, the mathematical theory of viability proves to be relevant. The viability domain under consideration encompasses the set of initial conditions soil moisture and nitrogen content at the time of sowing that allows, through fertigation, for the attainment of maximum biomass at harvest. More precisely, crops suffer neither water stress nor nitrogen stress, that is the trajectories $(S(t),N(t))$ remain in the set $E:=\left\{(S,N); S\geq S^* \text{ and } \frac{N}{S}\geq\eta_c\right\}$. Then the maximal biomass production is guaranteed. The domain E (see Fig.d) is considered "viable" if, starting from any initial condition (represented by t_0 , S_0 and N_0), we can always find a way to control the system (through the variable u) so that the system's behavior remains within acceptable limits (stays inside the domain E) throughout the entire time period from t_0 to T. Our viability analysis, utilizing the model,

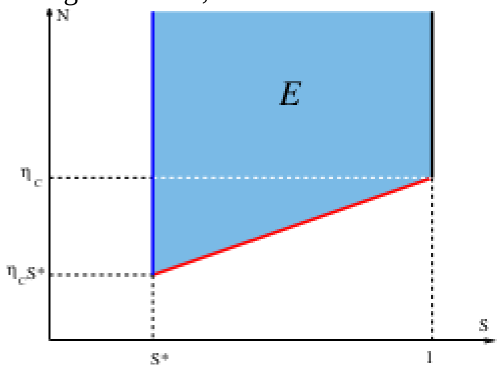

Fig.d

enables precise delineation of the permissible ranges for nitrogen concentration in the irrigation water and the maximum irrigation rate necessary to uphold the viability property. The viability thresholds identified by the model delineate the minimum input requirements to achieve maximum biomass at harvesting time, while avoiding unnecessary nitrogen accumulation in the soil. Subsequently, we explore control strategies aimed at maximizing biomass (i.e., maintaining conditions within the viability domain) while minimizing total water consumption. This constitutes an optimal control problem under a viability constraint, for which we have identified a variety of markedly different control strategies as depicted follows depending on threshold values of the initial nitrogen concentrations $N_0^b > N_0^+ > N_0^\# > \eta_C$ (the existence of such values have been proved mathematically in [1]):

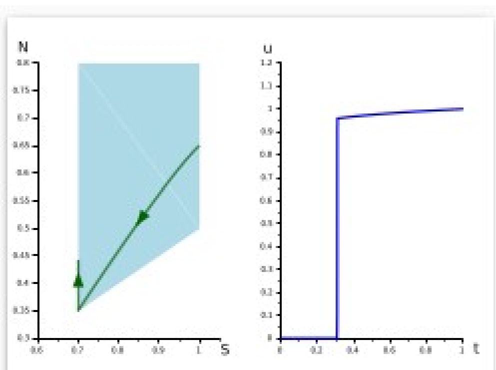

Fig1. Optimal strategy for $N_0 \geqslant N_0^b$

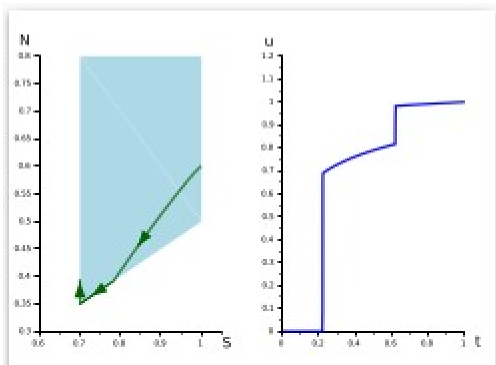

Fig2. Optimal strategy for $N_0 \in \left[N_0^+, N_0^b\right)$

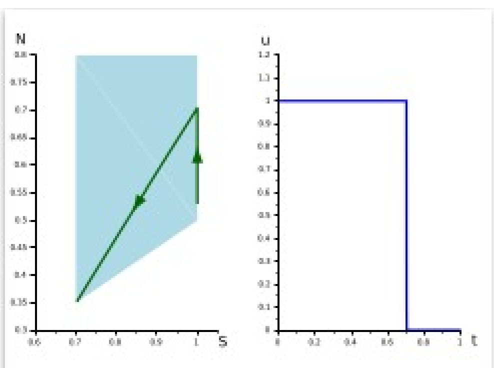

Fig3. Optimal strategy for $N_0 \in \left[\eta_C, N_0^\#\right)$

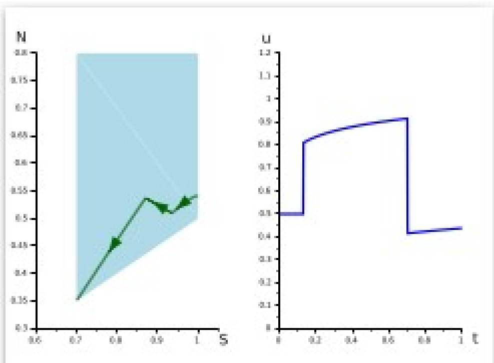

Fig4. Optimal strategy for $N_0 \in \left[N_0^\#, N_0^+\right)$

When the initial quantity of nitrogen is high $\left(N_0 \geqslant N_0^b\right)$ the optimal strategy consists in no irrigation up to the time the humidity threshold S^* is met and then keeping the humidity level exactly equal to this threshold up to the final time (see Fig1). For lower value of initial nitrogen $N_0^+ \leqslant N_0 \square \leqslant N_0^b$ the lack of irrigation causes crops to suffer from nitrogen stress before experiencing water stress and the preceding strategy is suboptimal. The optimal solution consists in keeping the system at the edge of this constraint until it reaches the humidity threshold and then keeping the humidity level exactly equal to this threshold (see Fig2). A surprising feature occurs when keeping the system at the edge of the nitrogen stress does not allow to reach the humidity threshold (because the time horizon as been reached before), for small initial content of nitrogen. Fig3 gives the case $\eta_C \leqslant N_0 \leqslant N_0^\#$, the optimal strategy consists in irrigating since the beginning to maintain the humidity level at his maximal level up to a precise time from which stopping the irrigation conducts the system to touch the nitrogen stress exactly at the terminal time. Finally, another non-intuitive feature occurs when the initial nitrogen level is between $N_0^\#$ and N_0^+ , the corner point of set E is reached precisely at the terminal time. The optimal strategy involves continuous irrigation following specific profiles, while avoiding the boundaries of set E. This research represents an initial step toward developing a decision support tool for irrigation management under water scarcity, particularly with the integration of reuse strategies (see [1]).

REFERENCE

1. Gildas Mahugnon Dadjo, Alain Rapaport, Jérôme Harmand, Rosane Ushirobira, Denis Efimov. Water-minimizing strategies under viability constraint for a crop fertigation model. 2023. Preprint https://hal.inrae.fr/hal-04267286.

2. Antoine Haddon, Alain Rapaport, Sébastien Roux, Jérôme Harmand. Model Based Optimization of Fertilization with Treated Wastewater Reuse. Advances in Water Resources, 2023, 181 (104561), ⟨10.1016/j.advwatres.2023.104561⟩.

3. Kenza Boumaza, Nesrine Kalboussi, Alain Rapaport, Sébastien Roux, Carole Sinfort. Optimal control of a crop irrigation model under water scarcity. Optimal Control Applications and Methods, 2021, 42 (6), pp.1612-1631. ⟨10.1002/oca.2749⟩.

4. N. Pelak, R. Revelli, and A. Porporato. A dynamical systems framework for crop models: Toward optimal fertilization and irrigation strategies under climatic variability. Ecological Modelling, 365:80–92, 2017

5. Aubin J.P, 2009. Viability Theory, Birkauser.