

Optimization of Irrigation with Treated Wastewater for Sustainable Olive Cultivation in Semi-Arid Areas

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ABSTRACT

In Tunisia, preserving conventional water resources is a national priority. To address drought conditions, the use of non-conventional water sources such as Treated Wastewater (TWW) for olive irrigation has become crucial. This work focuses on optimizing irrigation and nitrogen fertilization strategies to maximize olive yields while minimizing the environmental impact of using TWW in agriculture.

To achieve this goal, a dynamic crop model was developed, based on the "ToyCrop" model [1] and Pelak et al. [2], to describe the interaction between three main components: soil moisture content $S(t)$, total soil nitrogen content $N(t)$, and olive production $B(t)$. The model operates on a daily scale over a growing season.

• Water Balance :

The relative water Balance in the root zone (dimensionless between 0 and 1) is modeled as a balance between gains from precipitation R and irrigation I , and losses mainly due to soil evaporation E , crop transpiration T , and the combined rate of runoff and percolation Q_p [3]:

$$\frac{dS}{dt} = \frac{1}{\alpha Z} (R(t) + I(t) - T(t, S) - E(t, S) - Q_p(t, S)) \quad (1)$$

where αZ is the field capacity, with α being the soil porosity and Z the root depth. The transpiration and evaporation rates are described by the following equations:

$$T(t, S) = K_S(S) \phi(t) ET_0(t) \quad (2)$$

$$E(t, S) = K_R(S) (1 - \phi(t)) ET_0(t) \quad (3)$$

$$\phi(t) = \phi_{olive}(t) + \phi_{olive_tree}(t) \quad (4)$$

$$ET_0(t) = K_t (45.7 T_m + 813) p/100 \quad (5)$$

• Limiting Function :

The K_S function captures the plant's stomatal response to soil moisture, while K_R modulates evaporation according to the soil hygroscopic point S_h , below which no moisture loss occurs: :

$$K_S = \begin{cases} 0, & S \leq S_w \\ \frac{S - S_w}{S^* - S_w}, & S_w < S \leq S^* \\ 1, & S > S^* \end{cases} \quad (6)$$

$$K_R = \{0, S \leq S_h \frac{S - S_h}{1 - S_h}, S > S_h\} \quad (7)$$

- **Soil Nitrogen Balance**

The nitrogen balance evaluates nutrient inputs and outputs. Inputs come from fertigation, while the main extraction sources are leaching L_N and plant uptake U for crop production :

$$\frac{dN}{dt} = C_N^{in}(t)I(t) - U(t, N, S) - L_N(t, N, S) \quad (8)$$

$$U(t, S) = \frac{1}{\alpha Z} T(t, S) f\left(\frac{N}{S}\right) \quad (9)$$

- **Biomass Production**

The model assumes that biomass production is proportional to olive transpiration T_{olive}^* , with growth restrictions due to water and nitrogen limitations:

$$\frac{dB}{dt} = W^* T_{olive}^*(t, S) f\left(\frac{N}{S}\right) \quad (10)$$

where W^* is the daily normalized water productivity.

- **Viability Analysis**

Viability analysis addresses the dilemma of nitrogen concentration when making irrigation decisions. While water supply benefits crops, it can dilute soil nitrogen concentration, penalizing the plants [4]. This dilemma is especially relevant in the context of using recycled water, which can also provide nitrogen. To better understand this trade-off between water and nitrogen, we propose an approach based on viability theory rather than pure optimization.

We first formulate a model defining the constraints to be met during the growing season concerning soil moisture and nitrogen concentration to ensure optimal biomass production at harvest. The viability analysis examines how to keep the system's state, represented by soil moisture $S(t)$, nitrogen content $N(t)$, and produced biomass $B(t)$, within the constraint domain defined by:

$$K = \left\{ (S, N) ; S \geq S^* \text{ and } \frac{N}{S} \geq \eta_c \right\} \quad (15)$$

At any time $t \in [0, t_f]$ with a control $I(\cdot)$. The maximum irrigation flow rate I_{max} plays a key role in preventing the trajectory of variables $S(t)$ and $N(t)$ from exceeding the limits $S=S^*$ et $N = \eta_c S$ defined in the constraint domain K . A geometric condition to stay within this domain is to determine a control value $I(t)$ on the boundaries of K such that the velocity vector remains in this set at all times. This imposes a condition on I_{max} , called a "viability condition".

For an (S_0, N_0) , null control $I = 0$ (no irrigation) can be considered, and by integration, one determines whether the trajectory first touches the $S=S^*$ (water stress) or the $N/S=\eta_c$ (Nitrogen stress) boundary. Depending on the first boundary reached, the dominant stress, hydric or nitrogen, is identified. Finally, by integrating the control $I(t)$ over time, we obtain the amount of water needed to ensure that the system remains in the K region, thus ensuring maximum biomass production at harvest. This analysis was performed using model parameters calibrated for olive trees in Msaken, Tunisia.

- **Model Calibration**

The model calibration required collecting data from various sources such as local farmers, policymakers, field observations, as well as field and laboratory experiments. The data used for model calibration is summarized in Table 1. The model parameters, presented in Table 2, were estimated using the least squares method by fitting

experimental data through the MATLAB "fminsearch" optimization function. The calibration results showed a satisfactory fit of the model to field data.

Tableau 1. Soil Parameters

Parameter	Name	Value	Units	Source
S^*	Point of incipient stomatal closure	0.62	-	Soil analysis
S_w	Wilting point	0.02	-	Assumption
S_h	Hygroscopic point	0.02	-	Soil analysis
Z	Root Depth	0.8	m	ERT method
α	Soil porosity	0.21	-	Soil analysis

Tableau 2. Calibrated Parameters

Parameter	Name	Value	Units
η_c	Maximum N concentration taken up	0.047	Kg N/m ³
w^*	Normalized daily water productivity	5539.8	Kg B/m ² /day
k_L	Saturated hydraulic conductivity	15.25	m/d
d_L	Leakage parameter	9.03	-

Figure 1 shows the model results compared to experimental data. The results reveal a good fit between the model and experimental data. Indeed, the olive biomass production simulated by the model is very similar to the field production. Moreover, the nitrogen content determined in the laboratory closely matches the model results.

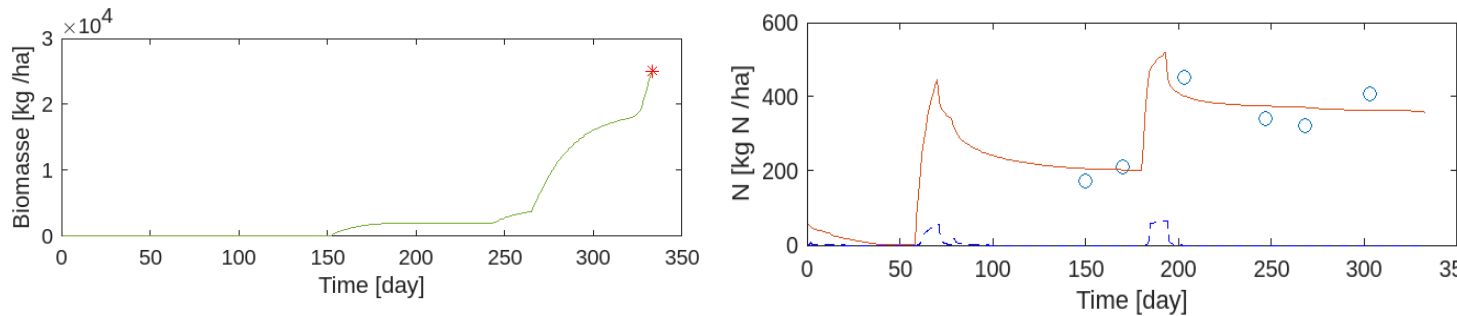


Figure 1 : Résultat de Calibration de modèle de culture

● **Viability Analysis Results**

The viability analysis shows that water stress is the main limiting factor for biomass production, greatly surpassing nitrogen stress. The system's trajectory remains on the water stress boundary $S = S^*$ without ever reaching the nitrogen stress limit. With irrigation using treated wastewater, the amount of nitrogen increases with irrigation, thus avoiding nitrogen stress.

The recommended strategy to minimize irrigation water while maintaining maximum biomass involves two phases: first, no irrigation until water stress appears, then minimal irrigation to keep the trajectory on the water stress boundary. The theoretical values proposed by the viability analysis include a maximum irrigation rate of 5.77 m³/day/ha and a total annual water requirement of 1240 m³/ha.

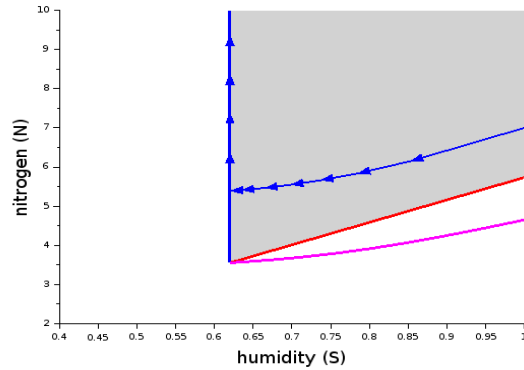


Figure 2 : Viability Analysis Results

Conclusion

This study highlights the crucial role of mathematical modeling as a decision-support tool for local stakeholders, facilitating informed decision-making and encouraging the adoption of treated wastewater reuse practices by farmers. Moreover, it emphasizes the feasibility of using treated wastewater to improve agricultural productivity, provided that appropriate safety measures are implemented to mitigate environmental risks.

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